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Three Dimensional Vibrations of Thermoporoelastic Solids with Two Temperatures

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Abstract

Three dimensional vibrations of thermoporoelastic solids with two temperatures are investigated in the framework of Biot's theory. Equations of motion are derived in the presence of two temperatures. Frequency equation is obtained. Frequency against wavenumber and temperature parameter is computed for two poroelastic solids. As the wavenumber increases frequency increases in two similar materials wherein solid part is sandstone and fluid parts are different. Frequency of one material values are greater than that frequency of other material. This is due to the influence of fluid part present in the materials.

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Keywords: Thermoporoelasticity; Two temperatures; Frequency equation; Wavenumber; Frequency.

1. Introduction

Wave propagation problems in thermoporoelastic solids have wide applications in various fields such as Engineering, Geophysics and Biological Sciences. The soft tissues in biological bodies are treated as a thermoporoelastic media. On a theory of heat conduction involving two temperatures is studied by Chen and Gurtin [1]. Chen et al., [2] investigated on the thermodynamics of non-simple elastic materials with two temperatures. The two temperature theory involves a material parameter $a > 0.$ The two temperature model has been widely used to

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predict the electron and phonon temperature distributions in ultra short laser processing of metals. Kumar and Mukhopadhyay [3] investigated effects of thermal relaxation time on plane wave propagation under two temperature thermoelasticity. In this paper transverse waves is unaffected due to the presence of thermal field whereas the dilatational waves is coupled with the thermal field. Warren and Chen [4] studied wave propagation in two temperature theory of thermoelasticity. Wave propagation in temperature rate dependent thermoelasticity with two temperatures is studied by Sachin Kaushal et al., [5]. Employing the Biot's theory, [6], Xiang-YuLi et al., [7] investigated general steady state solutions for transversely isotropic thermoporoelastic media in three dimensions and its applications. Wave propagation in thermoelastic saturated porous medium is studied by Sharma [8]. Hany Sherief and Eman M. Hussein [9] presented a mathematical model for short time filtration in poroelastic media with thermal relaxation and two temperatures. In the said paper, governing equations are derived for a mathematical model of generalized thermoelasticity in poroelastic media with thermal relaxation and two temperatures. The interaction between the processes of elasticity and heat in poroelastic material allowed for finite speeds of propagation of waves. Interaction due to expanding surface loads in thermoporoelastic medium is analysed by Rajneesh Kumar et al., [10]. Bao-Shangzhao, and Gui Xian Lu [11] discussed general steady state solution for thermoporoelastic material. However to the best of author's knowledge three dimensional vibrations of thermoporoelastic solids with two temperatures are not yet investigated. Therefore in this paper same is investigated in the framework of Biot's theory. The pertinent equations of motion are derived in the presence of two temperatures. Frequency is computed as a function of wavenumber for fixed two temperature parameter. Frequency is computed for two types of thermoporoelastic materials and then discussed.

This paper is organized as follows. In section 2, governing equations and solution of the problem are given. Numerical results and particular case are described in section 3. Finally, conclusions are given in section 4.

2. Governing Equations and Solution of the Problem

Consider thermoporoelastic solid in the Cartesian coordinate system in the absence of body forces [5, 6] are as follows:

$$\frac{\partial\sigma_{xx}}{\partial x}+\frac{\partial\sigma_{xy}}{\partial y}+\frac{\partial\sigma_{xz}}{\partial z}=\frac{\partial^2}{\partial t^2}(\rho_{11}u+\rho_{12}U),$$

$$\frac{\partial\sigma_{yx}}{\partial x}+\frac{\partial\sigma_{yy}}{\partial y}+\frac{\partial\sigma_{yz}}{\partial z}=\frac{\partial^2}{\partial t^2}(\rho_{11}v+\rho_{12}V),$$

$$\frac{\partial\sigma_{zx}}{\partial x}+\frac{\partial\sigma_{zy}}{\partial y}+\frac{\partial\sigma_{zz}}{\partial z}=\frac{\partial^2}{\partial t^2}(\rho_{11}w+\rho_{12}W),$$

$$\frac{\partial s}{\partial x}=\frac{\partial^2}{\partial t^2}(\rho_{12}u+\rho_{22}U),$$

$$\frac{\partial s}{\partial y}=\frac{\partial^2}{\partial t^2}(\rho_{12}v+\rho_{22}V),$$

$$\frac{\partial s}{\partial z}=\frac{\partial^2}{\partial t^2}(\rho_{12}w+\rho_{22}W), \tag{1}$$

$$K\nabla^2\varphi=\rho c_v(1+\tau_0\frac{\partial}{\partial t})\frac{\partial T}{\partial t}+\beta T_0(\frac{\partial}{\partial t}+\tau_0\frac{\partial^2}{\partial t^2})\nabla.u, \tag{2}$$

$$T=(1-a\nabla^2)\phi \tag{3}$$

In the above, eq. (1) pertaining to Biot's theory of isotropic poroelastic solids, whereas eq. (2) and eq. (3) governs heat conduction [5] and $\nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}$, ρ is the mass density, c_v is the specific heat capacity and K is the thermal conductivity, T_0 is the reference temperature, τ_0 is the relaxation time, a is the two temperature parameter, $\rho_{11}, \rho_{12}, \rho_{22}$ are the mass coefficients, (u,v,w) and (U,V,W) are the displacements of solid and fluid. s is the fluid pressure, b is the dissipation coefficient, σ_{ij} are the stress components are given by [5,6]

$$\sigma_{ij} = 2Ne_{ij} + (Ae + Q\varepsilon)\delta_{ij} - \beta T\delta_{ij},$$
$$s = Qe + R\varepsilon. \tag{4}$$

In eq. (4), e_{ij}'s are strain components, A, N, Q, R are poroelastic constants, β is the thermal stress, T is the temperature, e and ε are dilatations of solid and fluid. Substitution of eq. (3), eq. (4) in eq. (1) and eq. (2) and using the strain displacements relation [6] we get the equations of motion in the presence of two temperatures as follows:

$$N\nabla^2 u + (A+N)\frac{\partial e}{\partial x} + Q\frac{\partial \varepsilon}{\partial x} - \beta\frac{\partial(1-a\nabla^2)\phi}{\partial x} = \frac{\partial^2}{\partial t^2}(\rho_{11}u + \rho_{12}U),$$
$$N\nabla^2 v + (A+N)\frac{\partial e}{\partial y} + Q\frac{\partial \varepsilon}{\partial y} - \beta\frac{\partial(1-a\nabla^2)\phi}{\partial y} = \frac{\partial^2}{\partial t^2}(\rho_{11}v + \rho_{12}V),$$
$$N\nabla^2 w + (A+N)\frac{\partial e}{\partial z} + Q\frac{\partial \varepsilon}{\partial z} - \beta\frac{\partial(1-a\nabla^2)\phi}{\partial z} = \frac{\partial^2}{\partial t^2}(\rho_{11}w + \rho_{12}W),$$
$$Q\frac{\partial e}{\partial x} + R\frac{\partial \varepsilon}{\partial x} = \frac{\partial^2}{\partial t^2}(\rho_{12}u + \rho_{22}U),$$
$$Q\frac{\partial e}{\partial y} + R\frac{\partial \varepsilon}{\partial y} = \frac{\partial^2}{\partial t^2}(\rho_{12}v + \rho_{22}V),$$
$$Q\frac{\partial e}{\partial z} + R\frac{\partial \varepsilon}{\partial z} = \frac{\partial^2}{\partial t^2}(\rho_{12}w + \rho_{22}W),$$
$$K\nabla^2\phi = \rho c_v(1+\tau_0\frac{\partial}{\partial t})\frac{\partial(1-a\nabla^2)\phi}{\partial t} + \beta T_0(\frac{\partial}{\partial t} + \tau_0\frac{\partial^2}{\partial t^2})(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z}). \tag{5}$$

Now we can assume the solution of the eq. (5) in the following form [12]

$$u(x,y,z) = C_1 e^{j\omega t - j(k_1x + k_2y + k_3z)},$$
$$v(x,y,z) = C_2 e^{j\omega t - j(k_1x + k_2y + k_3z)},$$
$$w(x,y,z) = C_3 e^{j\omega t - j(k_1x + k_2y + k_3z)},$$
$$U(x,y,z) = C_4 e^{j\omega t - j(k_1x + k_2y + k_3z)},$$

$$V(x,y,z) = C_5 e^{j\omega t - j(k_1x + k_2y + k_3z)},$$
$$W(x,y,z) = C_6 e^{j\omega t - j(k_1x + k_2y + k_3z)},$$
$$\phi(x,y,z) = C_1 e^{j\omega t - j(k_1x + k_2y + k_3z)}. \tag{6}$$

In eq. (6), $C_1, C_2, C_3, C_4, C_5, C_6, C_7$ are arbitrary constants, j is the complex unity and $k_i\,(i = 1, 2, 3)$ is the wave number in the i^{th} direction such that the wave number $k = \sqrt{k_1^2 + k_2^2 + k_3^2}$. Substituting eq. (6) in the eq. (5), we obtain

$$(N(k_1^2 + k_2^2 + k_3^2) + (A+N)k_1^2 - \omega^2\rho_{11})C_1 + (A+N)k_1k_2C_2 + (A+N)k_2k_3C_3 + (Qk_1^2 - \omega^2\rho_{12})C_4$$
$$+ Qk_1k_2C_5 + Qk_1k_3C_6 + \beta j(-k_1 - a(k_1^3 + k_1k_2^2 + k_1k_3^2))C_7 = 0,$$

$$(A+N)k_1k_2C_1 + (Pk_2^2 + N(k_1^2 + k_3^2) - \omega^2\rho_{11})C_2 - (A+N)k_2k_3C_3 + Qk_1k_2C_4 + (Qk_2^2 - \omega^2\rho_{12})C_5$$
$$+ Qk_2k_3C_6 + \beta j(-k_2 - a(k_1k_2 + k_2^2 + k_2k_3))C_7 = 0,$$

$$(A+N)k_1k_3C_1 + (A+N)k_1k_3C_2 + (Pk_3^2 + N(k_1^2 + k_2^2) - \omega^2\rho_{11})C_3 + Qk_1k_3C_4 + Qk_2k_3C_5$$
$$+ \beta j(-k_3^2 - a(k_1^2k_3 + k_2^2k_3 + k_3^2))C_7 = 0,$$

$$(Qk_1^2 - \omega^2\rho_{12})C_1 + Qk_1k_2C_2 + Qk_1k_3C_3 + (Rk_1^2 - \omega^2\rho_{22})C_4 + Rk_1k_2C_5 + Rk_1k_3C_6 = 0,$$

$$Qk_1k_2C_1 + (Qk_2^2 - \omega^2\rho_{12})C_2 + Qk_2k_3C_3 + Rk_1k_2C_4 + (Rk_2^2 - \omega^2\rho_{22})C_5 + Rk_2k_3C_6 = 0,$$

$$Qk_1k_3C_1 + Qk_2k_3C_2 + (Qk_3^2 - \omega^2\rho_{12})C_3 + Rk_1k_3C_4 + Rk_2k_3C_5 + (Rk_3^2 - \omega^2\rho_{22})C_6 = 0,$$

$$\frac{\omega^2\beta T_0\tau_0k_3}{j}C_3 + (Kk_3^2 - \rho c_v\omega^2\tau_0 - \rho c_v a\omega^2k_3^2\tau_0)C_7 = 0. \tag{7}$$

3. Numerical results

For the numerical results, we consider the wave propagation in the z-direction. In this case $k_1 = k_2 = 0$ and the eqs. (7) reduce to

$$\begin{bmatrix} A_{11} & 0 & 0 \\ 0 & A_{22} & 0 \\ 0 & 0 & A_{33} \end{bmatrix}\begin{bmatrix} C_1 \\ C_2 \\ C_3 \end{bmatrix} = 0, \tag{8}$$

where

$$A_{11} = Nk_3^2 - N\frac{\omega^2}{V_s^2},$$

$$A_{22} = Nk_3^2 - N\frac{\omega^2}{V_s^2},$$

$$\begin{aligned}A_{33} ={}& RKk_3^4\omega^2\rho_{11} - R\omega^2k_3^2\rho_{12}\rho c_v\omega^2\tau_0 - R\omega^4ak_3^4\rho_{11}\tau_0 - \omega^4\rho_{11}\rho_{22}Kk_3^2 + \omega^6\rho_{11}\rho_{22}\rho c_v\tau_0 + \omega^4\rho_{11}\rho_{12}\rho c_v ak_3^2\tau_0 \\ &- PRKk_3^6 + PRk_3^4\rho c_v\omega^2\tau_0 + PR\rho c_v a\omega^2k_3^6\tau_0 + PKk_3^4\omega^2\rho_{22}\tau_0 - Pk_3^4\omega^4\rho_{22}\rho c_v a\tau_0 - 2QKk_3^4\omega^2\rho_{12} \\ &+ 2Qk_3^2\omega^4\rho_{12}\tau_0 + 2Qk_3^4\omega^4\rho c_v a\tau_0 + Kk_3^2\omega^4\rho_{12} - \rho c_v\omega^6\rho_{12}^2\tau_0 - \rho c_v a\omega^6k_3^2\tau_0\rho_{12}^2 + Q^2Kk_3^2 - Q^2k_3^2\rho c_v\omega^2\tau_0 \\ &- Qk_3^6\rho c_v a\omega^2\tau_0 - R\beta^2\omega^2T_0\tau_0k_3^2 - R\beta^2\omega^2aT_0\tau_0k_3^2 + \beta^2\omega^4T_0\tau_0\rho_{22}k_3^2 + \beta^2\omega^4aT_0\tau_0k_3^2\rho_{22}.\end{aligned}$$

In the above, V_s is the shear wave velocity [6]. For a non-trivial solution, the determinant of above coefficient matrix is zero. This leads to the frequency equation:

$$\begin{vmatrix} A_{11} & 0 & 0 \\ 0 & A_{22} & 0 \\ 0 & 0 & A_{33} \end{vmatrix} = 0 \tag{9}$$

The frequency equation (9) is investigated for particular solids namely material-1 is sandstone saturated with kerosene while material-2 is sandstone saturated water are given by [13, 14, 15]

Material-1

$$\begin{aligned}&A = 0.4436\times10^{10}\,N/m^2,\quad N = 0.2765\times10^{10}\,N/m^2,\quad Q = 0.07635\times10^{10}\,N/m^2,\\ &R = 0.0326\times10^{10}\,N/m^2, \rho_{11} = 1.926137\times10^3\,kg/m^3,\quad \rho_{12} = -0.002137\times10^3\,kg/m^3,\\ &\rho_{22} = 0.21537\times10^3\,kg/m^3, \beta = 3.2\times10^{-4}1/^0k,\quad K = 0.13\omega/m^0k, \rho c_v = 1.67\times10^6\,J/m^3k.\end{aligned} \tag{10}$$

Material-2

$$\begin{aligned}&A = 0.306\times10^{10}\,N/m^2,\quad N = 0.922\times10^{10}\,N/m^2, Q = 0.013\times10^{10}\,N/m^2,\\ &R = 0.0637\times10^{10}\,N/m^2, \rho_{11} = 1.90302\times10^3\,kg/m^3,\quad \rho_{12} = 0, \rho_{22} = 0.2268\times10^3\,kg/m^3,\\ &\beta = 6.6\times10^{-5}1/^0k,\ K = 0.607\omega/m^0k, \rho c_v = 4.17\times10^6\,J/m^3k, T = 296^0k,\quad \tau_0 = 10^{-3}s.\end{aligned} \tag{11}$$

For a given material, the frequency equation (9) an implicit relation between frequency, wavenumber and two temperature parameter is obtained. Frequency is computed as a function of wavenumber for fixed two temperature parameter and the results are depicted in figure-1. Figure-1shows the plots of frequency against wavenumber. From this figure it is clear that material-2 values are greater than that of material-1 for the same value of two temperature parameter. As the wavenumber increases frequency increases for both material-1, and 2.

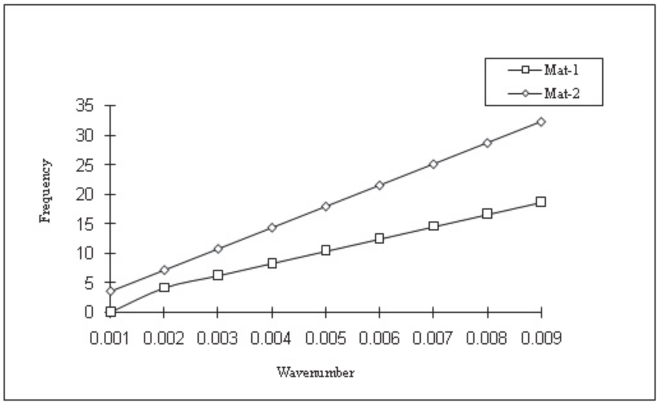

Fig. 1. Variation of frequency with the wavenumber

4. Conclusions

Employing Biot's theory, three dimensional vibrations in thermoporoelastic solids with two temperatures are investigated. Equations of motion are derived in the presence of two temperatures. Frequency against wavenumber is computed for fixed two temperature parameter. From the results, it is clear that sandstone saturated water values are greater than that of sandstone saturated kerosene for the same value of two temperature parameter. As the wavenumber increases frequency increases in two similar materials wherein solid part is sandstone and fluid parts are different. Frequency of one material values are greater than that frequency of other material. This is due to the influence of fluid part present in the materials.

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